Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2
Bronze Level B3

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
72	65	57	50	43	36

1. $$f(x) = x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b.

(5)

January 2011

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2. (a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(2)

(b) Solve, for $0 \leq x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(4)

January 2010

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3. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $5^x = 10$,

(2)

(b) $\log_3 (x - 2) = -1$.

(2)

May 2011

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4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

June 2010

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5. The third term of a geometric sequence is 324 and the sixth term is 96.

(*a*) Show that the common ratio of the sequence is $\frac{2}{3}$.

(2)

(*b*) Find the first term of the sequence.

(2)

(*c*) Find the sum of the first 15 terms of the sequence.

(3)

(*d*) Find the sum to infinity of the sequence.

(2)

June 2009

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6. Given that $2 \log_2 (x + 15) - \log_2 x = 6$,

(*a*) show that $x^2 - 34x + 225 = 0$.

(5)

(*b*) Hence, or otherwise, solve the equation $2 \log_2 (x + 15) - \log_2 x = 6$.

(2)

January 2013

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7. (*a*) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$,

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

January 2011

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8. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$.

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$.

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

January 2013

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9. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

January 2009

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10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

(*a*) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$.

(4)

(*b*) Hence find the maximum volume of the box.

(4)

(*c*) Use calculus to justify that the volume that you found in part (*b*) is a maximum.

(2)

January 2011

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$f(x) = x^4 + x^3 + 2x^2 + ax + b$	
	Attempting f(1) or f(–1).	M1
	$f(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$	A1 * cso (2)
(b)	Attempting f (–2) or f (2).	M1
	$f(-2) = 16 - 8 + 8 - 2a + b = -8$ $\{\Rightarrow -2a + b = -24\}$	A1
	Solving both equations simultaneously to get as far as $a = \ldots$ or $b = \ldots$	dM1
	Any one of $a = 9$ or $b = -6$	A1
	Both $a = 9$ and $b = -6$	A1 cso (5) **[7]**
2. (a)	$5\sin x = 1 + 2\left(1 - \sin^2 x\right)$	M1
	$2\sin^2 x + 5\sin x - 3 = 0$ **(*)**	A1cso (2)
(b)	$(2s - 1)(s + 3) = 0$ **giving** $s =$	M1
	$\left[\sin x = -3 \text{ has no solution}\right]$ **so** $\sin x = \frac{1}{2}$	A1
	$\therefore \quad x = 30,\ 150$	B1, B1ft (4) **[6]**
3. (a)	$5^x = 10$ and (b) $\log_3(x - 2) = -1$	
	$x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$	M1
	$x\ \{= 1.430676558...\} = 1.43\ (3\text{ sf})$	A1 cao (2)
(b)	$(x - 2) = 3^{-1}$	M1 oe
	$x\ \left\{= \frac{1}{3} + 2\right\} = 2\frac{1}{3}$	A1 (2) **[4]**
4. (a)	$(1 + ax)^7 = 1 + 7ax....$ or $1 + 7(ax)....$	B1
	$+ \dfrac{7 \times 6}{2}(ax)^2 + \dfrac{7 \times 6 \times 5}{6}(ax)^3$	M1
	$+ 21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$	A1
	$+ 35a^3x^3$ or $+35(ax)^3$ or $+35(a^3x^3)$	A1 (4)
(b)	$21a^2 = 525$	M1
	$a = \pm 5$	A1 (2) **[6]**

Question Number	Scheme	Marks
5. (a)	$324r^3 = 96$ or $r^3 = \dfrac{96}{324}$ or $r^3 = \dfrac{8}{27}$	M1
	$r = \dfrac{2}{3}$	A1cso
		(2)
(b)	$a\left(\dfrac{2}{3}\right)^2 = 324$ or $a\left(\dfrac{2}{3}\right)^5 = 96$ $a = \ldots$, 729	M1, A1
		(2)
(c)	$S_{15} = \dfrac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}}$, $= 2182.00\ldots$ (awrt 2180)	M1A1ft,
		(3)
(d)	$S_\infty = \dfrac{729}{1-\frac{2}{3}}$, $= 2187$	M1, A1
		(2) **[9]**
6. (a)	$2\log(x+15) = \log(x+15)^2$	B1
	$\log(x+15)^2 - \log x = \log\dfrac{(x+15)^2}{x}$	M1
	$2^6 = 64$ or $\log_2 64 = 6$	B1
	$\log_2\dfrac{(x+15)^2}{x} = 6 \Rightarrow \dfrac{(x+15)^2}{x} = 64$	M1
	$\Rightarrow x^2 + 30x + 225 = 64x$	
	or $x + 30 + 225x^{-1} = 64$	
	$\therefore x^2 - 34x + 225 = 0$ *	A1
		(5)
(b)	$(x-25)(x-9) = 0 \Rightarrow x = 25$ or $x = 9$	M1 A1
		(2) **[7]**

Question Number	Scheme	Marks
7. (a)	$3\sin^2 x + 7\sin x = \cos^2 x - 4$; $0 \le x < 360°$	
	$3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$	M1
	$4\sin^2 x + 7\sin x + 3 = 0$	A1 cso
		(2)
(b)	$(4\sin x + 3)(\sin x + 1)$ $\{= 0\}$	M1
	$\sin x = -\frac{3}{4}$, $\sin x = -1$	A1
	$(\lvert\alpha\rvert = 48.59\ldots)$	
	$x = 180 + 48.59$ or $x = 360 - 48.59$	dM1
	$x = 228.59\ldots$; $x = 311.41\ldots$	A1
	$\{\sin x = -1\} \Rightarrow x = 270$	B1
		(5)
		[7]
8. (a)	$y = 6 - 3x - \frac{4}{x^3}$	
	$\frac{dy}{dx} = -3 + \frac{12}{x^4}$ *or* $-3 + 12x^{-4}$	M1 A1
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = \ldots$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\left(\sqrt{2}\right)^4}$ or $-3 + 12\left(\sqrt{2}\right)^{-4} = 0$	A1
		(4)
(b)	$x = -\sqrt{2}$	B1
		(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	B1ft
		(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum	B1
	Maximum at P as $y'' < 0$	B1 cso
	Minimum at Q as $y'' > 0$	B1 cso
		(3)
		[9]

Question Number	Scheme	Marks
9. (a)	Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$	
	Or one of: $r = \dfrac{k}{k+4},\quad r = \dfrac{2k-15}{k},\quad r^2 = \dfrac{2k-15}{k+4}$,	M1
	Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$	
	$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$	M1, A1
	Proceed to $k^2 - 7k - 60 = 0$ (*)	A1
		(4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1
		(2)
(c)	Common ratio: $\dfrac{k}{k+4}$ or $\dfrac{2k\text{-}15}{k} = \dfrac{12}{16}\ \left(= \dfrac{3}{4}\ \text{or}\ 0.75\right)$	M1 A1
		(2)
(d)	$\dfrac{a}{1-r} = \dfrac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1
		(2) **[10]**
10. (a)	$V = 4x(5-x)^2 = 4x(25 - 10x + x^2)$	M1
	So, $V = 100x - 40x^2 + 4x^3$	A1
	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = 100 - 80x + 12x^2$	M1 A1cao
		(4)
(b)	$100 - 80x + 12x^2 = 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$	
	{As $0 < x < 5$} $x = \dfrac{5}{3}$	A1
	$x = \dfrac{5}{3},\quad V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$	dM1
	So $V = \dfrac{2000}{27} = 74\dfrac{2}{27} = 74.074\ldots$	A1
		(4)
(c)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = -80 + 24x$	M1
	When $x = \dfrac{5}{3},\quad \dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = -80 + 24\left(\dfrac{5}{3}\right)$	
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = -40 < 0 \quad \Rightarrow$V is a maximum	A1 cso
		(2) **[10]**

Examiner reports

Question 1

This question was accessible to the nearly all the candidates with the majority of them attempting to use the remainder theorem. In part (a), many candidates substituted $x = 1$ into the $\mathrm{f}(x)$ expression and were able to achieve the required $a + b = 3$. A few candidates, however, substituted $x = -1$ into $\mathrm{f}(x)$.

In part (b), most candidates attempted to find $\mathrm{f}(-2)$ and applied $\mathrm{f}(-2) = -8$ to give $16 - 8 + 8 - 2a + b = -8$. A significant minority of candidates incorrectly simplified $(-2)^4$ as -16 and a few candidates incorrectly set their $\mathrm{f}(-2)$ equal to 8 or even 0.

Poor manipulation was also a common feature, with some candidates simplifying $16 - 8 + 8 - 2a + b = -8$ incorrectly to give $-2a + b = 8$. The need to solve the two equations simultaneously was clearly understood and generally correctly applied, although those candidates who had made sign or manipulation slips earlier were unable to access the final two marks for finding both a and b.

A small minority of candidates attempted to use a method of long division in parts (a) and (b). The majority of these candidates usually failed to achieve a remainder in a and b which was independent of x. Some able candidates, however, handled long division with confidence and gained full marks in both parts of the question.

Question 2

(a) Most candidates correctly substituted $1 - \sin^2 x$ for $\cos^2 x$, but some lost the accuracy mark through incorrect manipulation of their equation or failure to put "equals zero".

(b) Most factorised or used the formula correctly and earned the first two marks. The most common errors again involved wrong signs. Most candidates correctly obtained the two answers 30 and 150 degrees. Some however gave the second angle as 210, others as 330 and another significant group gave three answers. Those who had made sign errors were able to get a follow through mark for giving a second angle consistent with their first.

Question 3

In part (a), the majority of candidates were able to use logs to correctly obtain 1.43, although some failed to round their answer to 3 significant figures as required by the question. It was common to see either method of $x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$. A few weaker candidates were able to achieve the correct answer by a method of trial and improvement.

About 60% of the candidates were able to answer part (b) correctly, with a small number offering no solution to this part. Although most candidates appreciated the need to remove logs, a number were unable deal with the -1, often rewriting $\log_3(x-2) = -1$ as $\log_3(x-2) = -\log_3(3)$ or $\log_3(x-2) = \log_3(-3)$ and then cancelling the logs from each side to get $x - 2 = -3$.

Another far too common response, showing a clear lack of understanding of the laws of logarithms, was to replace $\log_3(x-2)$ with $\log_3 x - \log_3 2$ and then $\log_3\left(\dfrac{x}{2}\right)$ or even

$\frac{\log_3 x}{\log_3 2}$. Those candidates who correctly removed the logarithm by writing $x - 2 = 3^{-1}$, usually achieved the correct answer.

Question 4

In part (a), most candidates exhibited understanding of the structure of a binomial expansion and were able to gain at least the method mark. Coefficients were generally found using the $(1 + x)^n$ binomial expansion formula, but Pascal's triangle was also popular. The correctly simplified third and fourth terms, $21a^2x^2$ and $35a^3x^3$, were often obtained and it was pleasing that $21ax^2$ and $35ax^3$ appeared less frequently than might have been expected from the evidence of previous C2 papers. Candidates tend to penalise themselves due to their reluctance to use brackets in terms such as $21(ax)^2$ and $35(ax)^3$.

Part (b) was often completed successfully, but some candidates included powers of x in their 'coefficients'. There is still an apparent lack of understanding of the difference between 'coefficients' and 'terms'. Although the question asked for the 'values' of a, some candidates gave only $a = 5$, ignoring the other possibility $a = -5$.

Question 5

There were many excellent solutions to this question. When problems did occur, these were frequently in part (a), where some candidates showed insufficient working to establish the given common ratio and others confused common ratio and common difference, treating the sequence as arithmetic.

Most of those who were confused in part (a) seemed to recover in part (b). In both part (a) and part (b), some candidates used the formula ar^{n-1} and others successfully used the method of repeatedly multiplying or dividing by the common ratio.

Formulae and methods for the sum to 15 terms and the sum to infinity were usually correct in parts (c) and (d). Just a few candidates found the 15th term instead of the sum in part (c) and just a few resorted to finding all 15 terms and adding.

Question 6

In part (a) logarithms were challenging for the less able candidates. Although many could apply the power rule correctly to obtain $2\log (x + 15) = \log (x + 15)^2$, some then proceeded to $\frac{\log(x+15)^2}{\log x} = 6$. Some candidates also erroneously started with $\frac{2\log(x+15)}{\log x} = 6$ or $2\log\left(\frac{x+15}{x}\right) = 6$ and were unable to gain much credit. The next stage was answered better and many candidates knew that to remove logs, 2^6 was required on the right hand side.

Part (b) involved solving the quadratic from Q6(a) and the majority opted to use factorisation successfully. Some chose to use the quadratic formula and were less successful, making arithmetic errors or using an incorrect formula.

Question 7

Most candidates were able to score both marks in part (a). Most candidates proceeded by replacing $1 - \sin^2 x$ for $\cos^2 x$. A few candidates, however, made algebraic errors or slips in rearranging the equation correctly into the result given.

The need to use the alternative form was understood in part (b) and most candidates made a valid attempt at factorisation, with correct factors being seen much more frequently than incorrect ones. Some candidates correctly wrote $(4\sin x + 3)(\sin x + 1) = 0$ and solved this incorrectly to give one of their solutions as $\sin x = \frac{3}{4}$. Of those candidates achieving the correct two values for $\sin x$ many only gave two correct solutions, usually 228.6 and 270 or 311.4 and 270. Sometimes extra incorrect solutions were given, usually 131.4 and/or 90. A small number of candidates found (270 ± their $|\alpha|$) rather than (180 + $|\alpha|$) and (360 – $|\alpha|$). Some candidates incorrectly stated that $\sin x - 1$had no solutions and a few gave their answers to the nearest degree. A significant number of candidates used a sketch of $\sin x$ to help them to correctly identify their answers.

Question 8

In part (a) the majority of candidates differentiated correctly and then either chose to solve $\frac{dy}{dx} = 0$ or substituted $x = \sqrt{2}$ to establish the turning point at P.

In part (b) many wrote down $x = -\sqrt{2}$ but also a significant number of candidates went back to the original equation in part (a) and attempted to find the other solution, with varying degrees of success.

The differentiation in part (c) was answered well and recovery was allowed from an incorrect derivative in part (a). There was a clear demand to establish the nature of the turning points at P and Q with justification. There were many cases where candidates made no reference to the fact that the sign of the second derivative was the determining factor and simply evaluated the second derivative at P and Q and stated whether they thought they were a maximum or minimum.

Question 9

Part (a) was a good discriminator. There were a few cases of "fudging" attempts to yield the printed answer using (k + 4)(2k – 15) = 0 or similar. Cancelling was often ignored by those using (k + 4) x (k/(k + 4))² = (2k – 15) resulting in cubic equations – generally incorrectly expanded.

Finding the printed answer in (b) was straightforward and most were successful at solving the quadratic equation. Some used verification and lost a mark.

Finding the common ratio in part (c) was answered well, though some candidates found r = 4/3 however.

The sum to infinity in (d) was answered well. Using 12 for "a" was the frequent error here.

Question 10

In part (a), most candidates expanded V to obtain a cubic equation of the correct form and then differentiated this to give the correct result. Occasional slips, usually with signs, appeared as did the loss of a term when squaring $(5 - x)^2$. A few candidates attempted to use the product rule but most of them made slips.

In part (b), nearly all candidates were able to put their answer from part (a) equal to 0 and many candidates obtained $x = \frac{5}{3}$ with most of them realising that $x = 5$ was outside the range. Unfortunately a significant number of candidates did not substitute their x-value into an expression for V in order to find the maximum volume. A significant minority of candidates tried to find the value of x which satisfied $\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = 0$.

In part (c), most candidates knew an appropriate method with almost all opting to find $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$. The final mark was often lost, however, due to candidates differentiating an incorrect $\frac{\mathrm{d}V}{\mathrm{d}x}$ or equating their second differential to zero or failing to evaluate the second differential, and then stating that this was negative which meant that the volume found in part (b) was maximum.

Statistics for C2 Practice Paper Bronze Level B3

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	7		83	5.81	6.84	6.62	6.30	5.69	5.08	4.34	3.21
2	6		82	4.93		5.82	5.53	5.02	3.83	2.88	1.56
3	4		76	3.05	3.95	3.83	3.50	3.13	2.74	2.37	1.65
4	6		73	4.37	5.96	5.78	5.32	4.73	4.01	3.20	1.60
5	9		81	7.30		8.75	8.35	7.86	7.12	6.29	3.96
6	7		78	5.47	7.00	6.76	6.17	5.25	4.29	3.63	1.89
7	7		78	5.43	6.93	6.59	6.02	5.28	4.27	3.33	1.92
8	9		73	6.53	8.81	8.32	7.25	6.22	5.10	3.98	2.26
9	10		65	6.54		9.04	7.18	5.71	4.50	3.47	1.96
10	10		68	6.81	9.81	9.02	7.27	5.94	4.66	3.68	2.32
	75		**75**	**56.24**		**70.53**	**62.89**	**54.83**	**45.60**	**37.17**	**22.33**